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Notes on supersymmetry enhancement of ABJM theory

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Abstract: We study the supersymmetry enhancement of ABJM theory. Starting from a $\mathcal{N}=2$ supersymmetric Chern-Simons matter theory with gauge group U(2)×U(2) which is a truncated version of the ABJM theory, we find by using the monopole operator that there is additional $\mathcal{N}=2$ supersymmetry related to the gauge group. We show this additional supersymmetry can combine with $\mathcal{N}=6$ supersymmetry of the original ABJM theory to an enhanced $\mathcal{N}=8$ SUSY with gauge group U(2)×U(2) in the case $k=1,2$. We also discuss the supersymmetry enhancement of the ABJM theory with U(N)×U(N) gauge group and find a condition which should be satisfied by the monopole operator.

Keywords: M-Theory, Conformal Field Models in String Theory

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JHEP08(2009)093

Contents

1 Introduction

There has been remarkable recent progress in understanding the worldvolume theory of coincident M2-branes. This was initiated by Bagger and Lambert [1] and Gustavsson [2] (BLG) who found an $\mathcal{N} = 8$ Chern-Simons matter theory based on 3-algebra. Under the assumption for Euclidean metric in the 3-algebra, the gauge group of the BLG theory is restricted to SO(4). So the BLG theory can be reformulated as an ordinary Chern-Simons gauge theory with SU(2)×SU(2) gauge group having opposite Chern-Simons levels k and $-k$ [3]. Inspired by BLG theory and subsequent developments, Aharony, Bergman, Jafferis, and Maldacena (ABJM) proposed $\mathcal{N} = 6$ Chern-Simons matter theory with U(N)×U(N) gauge group [4]. The ABJM theory is believed as a low energy effective theory of multiple M2-branes on orbifold $\mathbf{C}^4/\mathbf{Z}_k$. According to the developments related to the M2-brane effective actions, the Chern-Simons matter theories with higher number ($\mathcal{N} \geq 4$) of supersymmetry were also constructed [5–7].

The ABJM theory with gauge group SU(2)×SU(2) is equivalent to the BLG theory as proved in ref. [4]. So it has $\mathcal{N} = 8$ supersymmetry regardless the Chern-Simons level k. Unlike the SU(2)×SU(2) case, the ABJM theory has $\mathcal{N} = 6$ supersymmetry for generic k. It

JHEP08(2009)093

was conjectured, however, that the ABJM theory has the additional $\mathcal{N}=2$ supersymmetry and becomes $\mathcal{N}=8$ theory at $k=1,2$ [4].

The purpose of this paper is to find the additional $\mathcal{N}=2$ supersymmetry explicitly and prove the conjecture for supersymmetry enhancement in ABJM theory with U(1)×U(1) and U(2)×U(2) gauge groups. We also propose a general formulation for the additional supersymmetry in ABJM theory with U(N)×U(N) gauge group. To do so, we introduce a local operator $T^{ab}_{\hat{a}\hat{b}}$ (or $T^{\hat{a}\hat{b}}_{ab}$) in the supersymmetry transformation rules, where a, b and $\hat{a}, \hat{b}$ are the gauge indices of U$(N)_L$ and U$(N)_R$ gauge groups respectively. After some calculations we determine the condition for T, which gives the additional $\mathcal{N}=2$ supersymmetry. Since there are two gauge groups in ABJM theory, the matter fields are in bifundamental or anti-bifundamental representations, which are interchanged with each other with the action of T on these fields. For instance, a bifundamental scalar Y^A is changed to an anti-bifundamental scalar TY^A due to the index structure of T. Actually T corresponds to the monopole operator (often called 't Hooft operator), which was suggested in ref. [8]. For an explicit study of monopole operators in the ABJM theory and related topics, see [9–16]

It is interesting that the supersymmetry parameter for the additional $\mathcal{N}=2$ supersymmetry includes gauge indices and crucially depends on the gauge group of the theory. In this sense, the additional supersymmetry in ABJM theory is an exceptional one in supersymmetric gauge theories.

For U(1)×U(1) case, T becomes the abelian monopole operator as we will see in the subsection 2.1, and the additional supersymmetry is allowed for $k=1,2$ cases due to the orbifold structure of the transverse space. On the other hand, for SU(2)×SU(2) case, T is expressed as the product of the SU(2) invariant tensors ϵ^{ab} and $\epsilon_{\hat{a}\hat{b}}$, which are independent of the worldvolume coordinates, and the additional supersymmetry always exist for any value of k. Therefore the additional supersymmetry seems to be allowed only for $k=1,2$ cases in U(N)×U(N) or SU(N)×SU(N) ($N \geq 3$) gauge groups, which are composed of U(1) and SU(2) parts.

The rest of this paper is organized as follows. In section 2, we introduce a superconformal Chern-Simons matter theory which is a truncated version of the ABJM theory but has the same supersymmetry enhancement properties with minimal number of matter fields. We call this theory as the *minimal model*. The model has the same forms of the kinetic terms for scalars and fermions and the Chern-Simons terms. And the matter field part is composed of two complex scalars and fermions and so the fermionic and bosonic potentials are different from those of ABJM theory. We explicitly show $\mathcal{N}=2$ supersymmetry of the model having $U(1)_R$ symmetry and find the additional $\mathcal{N}=2$ supersymmetry for U(1)×U(1), SU(2)×SU(2), and U(2)×U(2) cases. In appendix A, we verify the supersymmetric invariance of the Lagrangian of the minimal model. In section 3, we prove the conjecture for the supersymmetry enhancement in ABJM theory for U(1)×U(1) and U(2)×U(2) cases at $k=1,2$, and suggest a possible supersymmetry transformation rules for the additional $\mathcal{N}=2$ supersymmetry and corresponding condition in T for the general U(N)×U(N) or SU(N)×SU(N) cases. In appendix B, we show that the procedure for the minimal model can also be applied to ABJM theory. We conclude in section 4 with brief summary and discussion.

Note Added. While this paper was being completed, a paper arXiv:0906.3568 [hep-th][17] appeared, which also deals with supersymmetry enhancement of ABJM theory with general gauge group based on 3-algebra.

2 Supersymmetry enhancement of a minimal model

Before taking into account the supersymmetry enhancement of ABJM theory, we consider supersymmetry enhancement of a minimal model, which is a $\mathcal{N}=2$ superconformal Chern-Simon matter theory and has the same supersymmetry enhancement behaviors with those of ABJM theory. The model has the same kinetic terms for scalars and fermions, Chern-Simons terms with gauge group U(N)×U(N)(or SU(N)×SU(N)) with the ABJM theory. However, the minimal model has two complex scalars and fermions with global SU(2)×U(1) symmetry, while ABJM theory has four complex scalars and fermions with global SU(4)×U(1) symmetry. So the fermionic and bosonic potentials of this $\mathcal{N}=2$ Chern-Simons theory are different from those of $\mathcal{N}=6$ ABJM theory.

Fields in the $\mathcal{N}=2$ minimal model are composed of two gauge fields A_μ and $\hat{A}_\mu$, two bifundamental bosonic fields Z^A and fermionic fields ψ_A with $A=1,2$, and their Hermitian conjugates $Z^\dagger_A$ and $\psi^{\dagger A}$ respectively. Z^A and $\psi^{\dagger A}$ with upper indices ($Z^\dagger_A$ and ψ_A with lower indices) are in the $\mathbf{2}$ ($\bar{\mathbf{2}}$) representation of the global SU(2). The gauge and matter fields have gauge group indices for U(N)×U(N) (or SU(N)×SU(N)) as $A^a{}_b$, $\hat{A}^{\hat{a}}{}_{\hat{b}}$, $Z^a{}_{\hat{b}}$, and $\psi^a{}_{\hat{b}}$. And the conjugate fields are represented as $Z^{\dagger\hat{a}}{}_b$ and $\psi^{\dagger\hat{a}}{}_b$. Then the action with global SU(2)×U(1) symmetry is given by[1]

$$S=\int d^3x\;(\mathcal{L}_0+\mathcal{L}_{\rm CS}-V_{\rm ferm}-V_{\rm bos}) \tag{2.1}$$

with

$$\begin{aligned}
\mathcal{L}_0&=\mathrm{tr}\left(-D_\mu Z^\dagger_A D^\mu Z^A+i\psi^{\dagger A}\gamma^\mu D_\mu\psi_A\right), && (2.2)\\
\mathcal{L}_{\rm CS}&=\frac{k}{4\pi}\,\epsilon^{\mu\nu\rho}\,\mathrm{tr}\left(A_\mu\partial_\nu A_\rho+\frac{2i}{3}A_\mu A_\nu A_\rho-\hat{A}_\mu\partial_\nu\hat{A}_\rho-\frac{2i}{3}\hat{A}_\mu\hat{A}_\nu\hat{A}_\rho\right), && (2.3)\\
V_{\rm ferm}&=\frac{2\pi i}{k}\mathrm{tr}\Big(Z^\dagger_A Z^A\psi^{\dagger B}\psi_B-Z^AZ^\dagger_A\psi_B\psi^{\dagger B}+2Z^AZ^\dagger_B\psi_A\psi^{\dagger B}-2Z^\dagger_AZ^B\psi^{\dagger A}\psi_B\Big), && (2.4)\\
V_{\rm bos}&=\frac{4\pi^2}{k^2}\mathrm{tr}\Big(Z^\dagger_AZ^AZ^\dagger_BZ^BZ^\dagger_CZ^C+Z^AZ^\dagger_AZ^BZ^\dagger_BZ^CZ^\dagger_C-2Z^AZ^\dagger_BZ^BZ^\dagger_AZ^CZ^\dagger_C\Big). && (2.5)
\end{aligned}$$

where the covariant derivatives are defined as

$$\begin{aligned}
D_\mu Z^A&=\partial_\mu Z^A+iA_\mu Z^A-iZ^A\hat{A}_\mu,\\
D_\mu Z^\dagger_A&=\partial_\mu Z^\dagger_A+i\hat{A}_\mu Z^\dagger_A-iZ^\dagger_A A_\mu.
\end{aligned} \tag{2.6}$$

[1] We choose (2+1)-dimensional gamma matrices which satisfy $\gamma^\mu\gamma^\nu=\eta^{\mu\nu}+\epsilon^{\mu\nu\rho}\gamma_\rho$ as $\gamma^0=i\sigma^2,\gamma^1=\sigma^1$, and $\gamma^2=\sigma^3$. The suppressed spinor indices are expressed by $\xi\chi\equiv\xi^\alpha\chi_\alpha$ and $\xi\gamma^\mu\chi=\xi^\alpha\gamma^{\mu\,\beta}_\alpha\chi_\beta$ for the two component spinors ξ and χ. The conventions of gauge indices for bosonic and fermionic fields are same as those in ref. [18].

JHEP08(2009)093

We can also obtain the action (2.1) by turning off two scalars and two fermions in the $\mathcal{N}=2$ superspace formalism for BLG theory given in ref. [18]. The F-term potentials vanish when we consider two complex fields only.

The action (2.1) is invariant under $\mathcal{N}=2$ supersymmetry transformation,

$$\begin{aligned}
\delta Z^A &= i\varepsilon^\dagger \epsilon^{AB}\psi_B,\\
\delta Z^\dagger_A &= i\epsilon_{AB}\psi^{\dagger B}\varepsilon,\\
\delta\psi_A &= \epsilon_{AB}D_\mu Z^B\gamma^\mu\varepsilon + \epsilon_{AB}N^B\varepsilon,\\
\delta\psi^{\dagger A} &= -\varepsilon^\dagger\epsilon^{AB}\gamma^\mu D_\mu Z^\dagger_B + \varepsilon^\dagger\epsilon^{AB}N^\dagger_B,\\
\delta A_\mu &= -\frac{2\pi}{k}\left(\varepsilon^\dagger\epsilon^{AB}\gamma_\mu\psi_B Z^\dagger_A + \epsilon_{AB}Z^A\psi^{\dagger B}\gamma_\mu\varepsilon\right),\\
\delta\hat{A}_\mu &= -\frac{2\pi}{k}\left(\varepsilon^\dagger\epsilon^{AB}Z^\dagger_A\gamma_\mu\psi_B + \epsilon_{AB}\psi^{\dagger B}\gamma_\mu Z^A\varepsilon\right),
\end{aligned}\tag{2.7}$$

where we define

$$N^A \equiv \frac{2\pi}{k}\left(Z^B Z^\dagger_B Z^A - Z^A Z^\dagger_B Z^B\right),\tag{2.8}$$

and ϵ^{AB} and ϵ_{AB} are the invariant tensors of the global SU(2) symmetry with $\epsilon^{12}=\epsilon_{12}=1$. ε and $\varepsilon^\dagger$ are the complex spinor parameter and its complex conjugate respectively. We prove the supersymmetry transformation rules (2.7) in appendix A.1.

The action (2.1) has the additional $\mathcal{N}=2$ supersymmetry depending on gauge group. As we will see later, the supersymmetry enhancement behaviors of the $\mathcal{N}=2$ minimal model are exactly same with those of ABJM theory. We find the additional supersymmetry for the model (2.1) with gauge groups, U(1)×U(1), SU(2)×SU(2), and U(2)×U(2) cases.

2.1 U(1)×U(1) case

In U(1)×U(1) case, we combine the two gauge fields A_μ and $\hat{A}_\mu$ into

$$A^\pm_\mu \equiv A_\mu \pm \hat{A}_\mu,\tag{2.9}$$

where A^+_μ does not interact with all matter fields and the corresponding flux is quantized via Chern-Simons terms, and A^-_μ is the $\mathrm{U}(1)_{A^-}$ (from now on, we denote it as $\mathrm{U}(1)_{A^-}$) gauge field which will be used inside the covariant derivatives (2.6). The matter fields $(Z^A,\psi^{\dagger A})$ in **2** representation of the global SU(2) have the $\mathrm{U}(1)_{A^-}$ charges $(+,-)$, while their Hermitian conjugates $(Z^\dagger_A,\psi_A)$ in $\bar{\mathbf{2}}$ representation have charges $(-,+)$. Since all matter fields are represented by complex numbers(not matrices) in this case, the fermionic and bosonic potentials in ABJM theory vanish (See eqs. (3.4) and (3.5)). Then the ABJM action with U(1)×U(1) gauge group is reduced to

$$S=\int d^3x\left(-D_\mu Z^\dagger_A D^\mu Z^A + i\psi^{\dagger A}\gamma^\mu D_\mu\psi_A + \frac{k}{4\pi}\epsilon^{\mu\nu\rho}A^-_\mu F^+_{\nu\rho}\right),\qquad \text{with } A=1,2,\tag{2.10}$$

where $D_\mu Z^A = \partial_\mu Z^A + iA^-_\mu Z^A$ and $F^+_{\mu\nu} = \partial_\mu A^+_\nu - \partial_\nu A^+_\mu$.

Since the covariant derivative in (2.10) depends only on A_μ^-, we can treat the field strength $F_{\mu\nu}^+$ as a fundamental field and introduce a dual scalar field $\tau(x)$. In order to do so, we have to add

$$S = \frac{1}{4\pi}\int d^3x\,\tau(x)\epsilon^{\mu\nu\rho}\partial_\mu F_{\nu\rho}^+ \tag{2.11}$$

to the action (2.10) [4, 19, 20], which represent the Bianchi identity for the gauge field strength $F_{\mu\nu}^+$, after we integrate out the auxiliary scalar field $\tau(x)$. Here $\tau(x)$ is 2π-periodic due to the flux quantization $\int d^3x\epsilon^{\mu\nu\rho}\,\partial_\mu F_{\nu\rho}^+ = 4\pi n$ with integer n. Then the equation of motion of $F_{\mu\nu}^+$ is given by

$$A_\mu^- = \frac{1}{k}\,\partial_\mu\tau. \tag{2.12}$$

The gauge transformation, $A_\mu^- \to A_\mu^- + \partial_\mu\Lambda$, implies that $\tau(x)$ is transformed as $\tau \to \tau + k\Lambda$. Though we fixed the gauge by taking $\tau = 0$, we can still perform gauge transformation with $\Lambda = \frac{2\pi}{k}$ from the periodic property of τ. This remaining symmetry implies

$$Z^A \sim e^{2\pi i/k}\,Z^A. \tag{2.13}$$

This means that the $\mathcal{N} = 2$ minimal model (2.10) with U(1)×U(1) gauge symmetry is reduced to the sigma model on $\mathbf{C}^2/\mathbf{Z}_k$ orbifold, while the original ABJM theory with U(1)×U(1) gauge group is reduced to the sigma model on $\mathbf{C}^4/\mathbf{Z}_k$ orbifold. The form of action (2.10) is equivalent to that of ABJM action with U(1)×U(1) gauge group, though the number of fields in (2.10) is half of that in ABJM theory. So many of the physical properties of the minimal model with U(1)×U(1) gauge group are similarly with those of ABJM theory with the same gauge group. Especially these two theories have the same supersymmetry enhancement properties.

Due to the abelian properties of matter fields in the minimal model with U(1)×U(1) gauge group, the supersymmetry transformation rules in (2.7) are reduced to

$$\begin{aligned}
\delta Z^A &= i\varepsilon^\dagger\epsilon^{AB}\psi_B,\\
\delta Z_A^\dagger &= i\epsilon_{AB}\psi^{\dagger B}\varepsilon,\\
\delta\psi_A &= \epsilon_{AB}D_\mu Z^B\gamma^\mu\varepsilon,\\
\delta\psi^{\dagger A} &= -\varepsilon^\dagger\epsilon^{AB}\gamma^\mu D_\mu Z_B^\dagger,\\
\delta A_\mu = \delta\hat{A}_\mu &= -\frac{2\pi}{k}\left(\varepsilon^\dagger\epsilon^{AB}\gamma_\mu\psi_B Z_A^\dagger + \epsilon_{AB}Z^A\psi^{\dagger B}\gamma_\mu\varepsilon\right).
\end{aligned} \tag{2.14}$$

As we see in the supersymmetry transformation rules of (2.14), the variations of bosonic field Z^A in $\mathbf{2}$ representation of the global SU(2) is proportional to the fermionic field ψ_A in $\bar{\mathbf{2}}$ representation. That is, the $\mathcal{N} = 2$ supersymmetry (2.14) relates the bosonic and fermionic fields with different global SU(2) representations. However, Z^A and ψ_A have the same $\mathrm{U}(1)_{A^-}$ charges.

Now we try to find the additional $\mathcal{N} = 2$ supersymmetry in the action (2.10). Differently from the $\mathcal{N} = 2$ supersymmetry given in (2.14), the additional supersymmetry relates the bosonic and fermionic fields with opposite $\mathrm{U}(1)_{A^-}$ charges but with the same global SU(2) representations. To compensate the $\mathrm{U}(1)_{A^-}$ charge differences, we need to include some abelian operator with two units $\mathrm{U}(1)_{A^-}$ charge in the supersymmetry transformation rules of the additional $\mathcal{N}=2$ supersymmetry, as suggested in ref. [8].

In order to find this kind of additional supersymmetry, we insert a local operator in the expected transformation rules, and find a condition for the operator to give supersymmetric invariance of the action (2.10). At first, we consider the following ansatz for the supersymmetry transformation rules,

$$\begin{aligned}
\delta_1 Z_A^\dagger &= i\tilde{\varepsilon}^\dagger \check{T}^* \psi_A, \\
\delta_1 \psi^{\dagger A} &= \tilde{\varepsilon}^\dagger \check{T}^* \gamma^\mu D_\mu^+ Z^A, \\
\delta_1 A_\mu^- &= 0,
\end{aligned} \tag{2.15}$$

where $\tilde{\varepsilon}^\dagger$ is the complex spinor parameter[2], $D_\mu^+ Z^A = \partial_\mu Z^A + iA_\mu^- Z^A$, and we introduce a worldvolume dependent operator $\check{T}^* = \check{T}^*(x)$ (complex conjugate of $\check{T}$) to compensate the $\mathrm{U}(1)_{A^-}$ charge differences in the relations (2.15). As will be explained later, $\check{T}$ becomes the monopole operator. Here, we denoted the monopole operator as $\check{T}$ in order to distinguish from the monopole operator T for the general U(N)$\times$ U(N) case. $\check{T}$ will also appear in U(2)$\times$ U(2) case. We only analyzed half of the terms in obtaining the supersymmetry transformation rules, since the remaining transformation rules are easily obtained by taking complex conjugate for (2.15). Applying the ansatz (2.15), we obtain

$$\begin{aligned}
\delta_1 \mathcal{L} =& -i\tilde{\varepsilon}^\dagger \gamma^\mu D_\mu^+ Z^A \gamma^\nu \left(\partial_\nu \check{T}^* - 2iA_\nu^- \check{T}^*\right)\psi_A \\
&- \frac{k}{4\pi}\epsilon^{\mu\nu\rho} A_\mu^- \partial_\nu \left(\frac{4\pi}{k}\,\tilde{\varepsilon}^\dagger \check{T}^* \gamma_\rho \psi_A Z^A - \delta_1 A_\rho^+\right),
\end{aligned} \tag{2.16}$$

up to total derivative terms. From the relation (2.16), we see that the $\mathcal{N}=2$ minimal model (2.10) is invariant under the additional $\mathcal{N}=2$ supersymmetry transformation (including the complex conjugate of (2.15)),

$$\begin{aligned}
\delta Z^A &= i\check{T}\psi^{\dagger A}\tilde{\varepsilon}, \\
\delta Z_A^\dagger &= i\tilde{\varepsilon}^\dagger \check{T}^* \psi_A, \\
\delta \psi_A &= -\check{T} D_\mu^- Z_A^\dagger \gamma^\mu \tilde{\varepsilon}, \\
\delta \psi^{\dagger A} &= \tilde{\varepsilon}^\dagger \check{T}^* \gamma^\mu D_\mu^+ Z^A, \\
\delta A_\mu = \delta \hat{A}_\mu &= \frac{2\pi}{k}\left(\check{T}\psi^{\dagger A}\gamma_\mu Z_A^\dagger \tilde{\varepsilon} + \tilde{\varepsilon}^\dagger \check{T}^* \gamma_\mu \psi_A Z^A\right),
\end{aligned} \tag{2.17}$$

where $D_\mu^- Z_A^\dagger = \partial_\mu Z_A^\dagger - iA_\mu^- Z_A^\dagger$ and $\check{T}$ satisfies the following differential equation

$$\partial_\mu \check{T} + 2iA_\mu^- \check{T} = 0. \tag{2.18}$$

Here we should notice, however, that the supersymmetry transformation rules (2.17) are not satisfied for all integer values of k, due to the orbifolding of matter fields given in (2.13).

[2] Throughout this paper, the complex spinor parameter $\tilde{\varepsilon}$ and its complex conjugate $\tilde{\varepsilon}^\dagger$ will be used to denote the additional $\mathcal{N}=2$ supersymmetry transformation rules.

From now on, we solve the equation (2.18) and try to figure out the properties of the local operator $\check{T}(x)$. Under the gauge transformation, $A^-_\mu \to A^-_\mu + \partial_\mu \Lambda$, the matter field (Z^A, ψ_A) with $+1$ $\mathrm{U}(1)_{A^-}$ charges, transform as $(Z^A, \psi_A) \to e^{-i\Lambda}(Z^A, \psi_A)$, while $(Z^\dagger_A, \psi^{\dagger A})$ with -1 $\mathrm{U}(1)_{A^-}$ charges transform as $(Z^\dagger_A, \psi^{\dagger A}) \to e^{+i\Lambda}(Z^\dagger_A, \psi^{\dagger A})$. Similarly from (2.18), we can easily see that $\check{T}$ transforms as $\check{T} \to e^{-2i\Lambda}\check{T}$ under the same gauge transformation, and so $\check{T}$ has $+2$ $\mathrm{U}(1)_{A^-}$ charge. We can also check of the $\mathrm{U}(1)_{A^-}$ charge of $\check{T}$ by solving the differential equation (2.18) directly. To guarantee the supersymmetry (2.14) over the whole worldvolume region, we consider the regular $\check{T}(x)$ only. Then the equation (2.18) implies that the $\mathrm{U}(1)_{A^-}$ gauge field A^-_μ is in pure gauge,

$$A^-_\mu = \frac{i}{2}\partial_\mu \ln \check{T}, \tag{2.19}$$

which corresponds to the result (2.12) by identifying

$$\check{T}(x) = \check{T}_0 e^{-2i\tau(x)/k} \tag{2.20}$$

with a constant $\check{T}_0$. Equivalently $\check{T}(x)$ can be expressed by the path independent local Wilson line [4, 11]

$$\check{T}(x) = \check{T}_0 e^{2i\int_x^\infty A^-_\mu dx^\mu}, \tag{2.21}$$

which transforms as $\check{T} \to e^{-2i\Lambda}\check{T}$ under the gauge transformation $A^-_\mu \to A^-_\mu + \partial_\mu\Lambda$. So we can also check that the operator $\check{T}$ has $+2$ $\mathrm{U}(1)_{A^-}$ charge. It was argued that attaching the local Wilson line to matter fields, we can change the $\mathrm{U}(1)_{A^-}$ charges of the fields and they are insensitive in the presence of the Wilson line [4, 11].

Due to the Chern-Simons term in the action (2.10), $\check{T}(x)$ becomes also the monopole operator (often called as 't Hooft operator [21]), which induces the magnetic flux from the position x. In three dimensional Chern-Simons matter theory, the Wilson line and 't Hooft operator are equivalent [22], as we have seen the equivalence between (2.21) and (2.20) which is the monopole operator.

Under a $\mathbf{Z}_k$ transformation for $\tau(x)$,

$$\tau(x) \to \tau(x) + 2\pi, \tag{2.22}$$

$\check{T}(x)$ given in (2.20) transforms as

$$\check{T}(x) \to e^{-4\pi i/k}\check{T}(x). \tag{2.23}$$

Therefore, the supersymmetry transformation rule is invariant under $\mathbf{Z}_k$ transformation only when $k = 1, 2$. It means that the orbifold ($k \geq 3$) structure of the transverse space breaks the supersymmetry invariance for the additional supersymmetry (2.17) of the action (2.10).

2.2 SU(2)×SU(2) case

The $\mathcal{N} = 2$ minimal model (2.1) with SU(2)×SU(2) gauge group has additional $\mathcal{N} = 2$ supersymmetry, in addition to the $\mathcal{N} = 2$ supersymmetry given in (2.7). Unlike the

U(1)×U(1) case which allows the additional $\mathcal{N} = 2$ supersymmetry for $k = 1, 2$ cases only, for SU(2)×SU(2) case the additional $\mathcal{N} = 2$ supersymmetry we will consider in this subsection does not depend on the value of the Chern-Simons level k. However, similarly to the case of U(1)×U(1), the supersymmetric variation of bosonic fields are proportional to fermionic fields with same representations of the global SU(2) but different gauge indices.

It turns out that the additional $\mathcal{N} = 2$ supersymmetry transformation rules of the matter fields in matrix notation are given by

$$\begin{aligned}
\delta Z^A &= i\tilde{\psi}^{\dagger A}\tilde{\varepsilon}, \\
\delta Z^\dagger_A &= i\tilde{\varepsilon}^\dagger \tilde{\psi}_A, \\
\delta\psi_A &= -D_\mu \tilde{Z}^\dagger_A \gamma^\mu \tilde{\varepsilon} - \tilde{N}^\dagger_A \tilde{\varepsilon}, \\
\delta\psi^{\dagger A} &= \tilde{\varepsilon}^\dagger \gamma^\mu D_\mu \tilde{Z}^A - \tilde{\varepsilon}^\dagger \tilde{N}^A, \\
\delta A_\mu &= \frac{2\pi}{k}\left(\tilde{\varepsilon}^\dagger Z^A \gamma_\mu \tilde{\psi}_A + \tilde{\psi}^{\dagger A}\gamma_\mu Z^\dagger_A \tilde{\varepsilon}\right), \\
\delta \hat{A}_\mu &= \frac{2\pi}{k}\left(\tilde{\varepsilon}^\dagger \gamma_\mu \tilde{\psi}_A Z^A + Z^\dagger_A \tilde{\psi}^{\dagger A}\gamma_\mu \tilde{\varepsilon}\right),
\end{aligned} \tag{2.24}$$

where $\tilde{\varepsilon}$ is the complex spinor parameter and we define the fields with *tilde* notation as follows,

$$\begin{aligned}
\tilde{Z}^{A\hat{a}}{}_a &\equiv \epsilon^{\hat{a}\hat{b}}\epsilon_{ab} Z^{Ab}{}_{\hat{b}}, & \tilde{Z}^{\dagger a}_{A\ \hat{a}} &\equiv \epsilon^{ab}\epsilon_{\hat{a}\hat{b}} Z^{\dagger\hat{b}}_{A\ b}, \\
\tilde{\psi}^{\hat{a}}_{Aa} &\equiv \epsilon^{\hat{a}\hat{b}}\epsilon_{ab}\psi^b_{A\hat{b}}, & \tilde{\psi}^{\dagger Aa}{}_{\hat{a}} &\equiv \epsilon^{ab}\epsilon_{\hat{a}\hat{b}}\psi^{\dagger A\hat{b}}{}_b, \\
\tilde{N}^{A\hat{a}}{}_a &\equiv \epsilon^{\hat{a}\hat{b}}\epsilon_{ab} N^{Ab}{}_{\hat{b}}, & \tilde{N}^{\dagger a}_{A\ \hat{a}} &\equiv \epsilon^{ab}\epsilon_{\hat{a}\hat{b}} N^{\dagger\hat{b}}_{A\ b}.
\end{aligned} \tag{2.25}$$

Here $\epsilon_{ab}(\epsilon^{\hat{a}\hat{b}})$ is the invariant tensor of the gauge group $\mathrm{SU}(2)_L(\mathrm{SU}(2)_R)$ and we explicitly denote the gauge indices for definiteness. In appendix A.2, we verify the invariance of the action (2.1) under the supersymmetry transformation (2.24).

Since there is no $\mathrm{U}(1)_{A^-}$ gauge symmetry in SU(2)×SU(2) case, the sypersymmetry transformation rules (2.24) do not include the monopole operators which do not allow the additional $\mathcal{N} = 2$ supersymmetry for $k > 2$ cases. So the minimal model with SU(2)×SU(2) gauge group has $\mathcal{N} = 4$ supersymmetry for arbitrary value of k.

2.3 U(2)×U(2) with both cases united

We investigated the supersymmetry enhancement of the $\mathcal{N} = 2$ minimal model (2.1) in the previous subsections. In U(1)×U(1) case, the $\mathcal{N} = 2$ supersymmetry (2.15) is enhanced to $\mathcal{N} = 4$ supersymmetry at $k = 1, 2$ only. In SU(2)×SU(2) case, however, the $\mathcal{N} = 2$ supersymmetry (2.15) is enhanced to $\mathcal{N} = 4$ supersymmetry regardless the value of k.

Similarly to the cases of U(1)×U(1) and SU(2)×SU(2), we find the additional $\mathcal{N} = 2$

supersymmetry transformation rules for U(2)×U(2) gauge group as follows

$$
\begin{aligned}
\delta Z^A &= i\tilde{\psi}^{\dagger A}\check{T}\tilde{\varepsilon}, \\
\delta Z^\dagger_A &= i\check{T}^*\tilde{\varepsilon}^\dagger\tilde{\psi}_A, \\
\delta\psi_A &= -D_\mu\tilde{Z}^\dagger_A\gamma^\mu\check{T}\tilde{\varepsilon} - \tilde{N}^\dagger_A\check{T}\tilde{\varepsilon}, \\
\delta\psi^{\dagger A} &= \check{T}^*\tilde{\varepsilon}^\dagger\gamma^\mu D_\mu\tilde{Z}^A - \check{T}^*\tilde{\varepsilon}^\dagger\tilde{N}^A, \\
\delta A_\mu &= \frac{2\pi}{k}\left(\check{T}^*\tilde{\varepsilon}^\dagger Z^A\gamma_\mu\tilde{\psi}_A + \tilde{\psi}^{\dagger A}\gamma_\mu Z^\dagger_A\check{T}\tilde{\varepsilon}\right), \\
\delta\hat{A}_\mu &= \frac{2\pi}{k}\left(\check{T}^*\tilde{\varepsilon}^\dagger\gamma_\mu\tilde{\psi}_A Z^A + Z^\dagger_A\tilde{\psi}^{\dagger A}\gamma_\mu\check{T}\tilde{\varepsilon}\right),
\end{aligned} \tag{2.26}
$$

where $\tilde{N}^A$ was defined in (2.25) and $\check{T}$ is the abelian monopole operator given in (2.20) or (2.21) with the $\mathrm{U}(1)_{A^-}$ gauge field, $A^-_\mu = \mathrm{tr}A_\mu - \mathrm{tr}\hat{A}_\mu$.

From (2.26), we can obtain the supersymmetry transformation (2.24) of SU(2)×SU(2) case by removing the monopole operator $\check{T}$. And also, we can obtain the supersymmetry transformation (2.17) of U(1)×U(1) case from (2.26) by regarding all fields in (2.26) as complex numbers without gauge indices. The reason is as follows. Dividing the gauge fields, A_μ and $\hat{A}_\mu$, into the trace part and traceless part, we can also decompose the action (2.1) with U(2)×U(2) gauge group into two parts with U(1)×U(1) and SU(2)×SU(2) gauge groups respectively. The gauge fields can be rewritten as

$$
A_\mu = B_\mu + C_\mu, \qquad \hat{A}_\mu = \hat{B}_\mu + \hat{C}_\mu,
$$

where $B_\mu = \mathrm{tr}(A_\mu)\frac{\mathbf{1}}{2}$ and $\hat{B}_\mu = \mathrm{tr}(\hat{A}_\mu)\frac{\mathbf{1}}{2}$ with 2×2 identity matrix $\mathbf{1}$. Using the trace property, we can rewrite the kinetic and Chern-Simons terms in (2.1) as

$$
\begin{aligned}
\mathcal{L}_0 =& \mathrm{tr}\left(-D_\mu Z^\dagger_A D^\mu Z^A + i\psi^{\dagger A}\gamma^\mu D_\mu\psi_A\right), \\
\mathcal{L}_{\mathrm{CS}} =& \frac{k}{4\pi}\left(B_\mu\partial_\nu B_\rho - \hat{B}_\mu\partial_\nu\hat{B}_\rho\right) \\
&+ \frac{k}{4\pi}\epsilon^{\mu\nu\rho}\,\mathrm{tr}\left(C_\mu\partial_\nu C_\rho + \frac{2i}{3}C_\mu C_\nu C_\rho - \hat{C}_\mu\partial_\nu\hat{C}_\rho - \frac{2i}{3}\hat{C}_\mu\hat{C}_\nu\hat{C}_\rho\right),
\end{aligned} \tag{2.27}
$$

where the covariant derivative is decomposed by

$$
D_\mu Z^A = \partial Z^A + i(B_\mu - \hat{B}_\mu)Z^A + iC_\mu Z^A - iZ^A\hat{C}_\mu. \tag{2.28}
$$

Since there is no potential for U(1)×U(1) case, we can think that the potentials in U(2)×U(2) case are decomposed into U(1)×U(1) part and SU(2)×SU(2) part already. From these reasons, we can read the supersymmetry transformation rules (2.17) and (2.24) from (2.26), and vice versa.

Since the supersymmetry transformation rules (2.26) include the monopole operator $\check{T}$, according to the discussion of subsection 2.1 the supersymmetry of the $\mathcal{N} = 2$ minimal model with U(2)×U(2) gauge group is enhanced to $\mathcal{N} = 4$ for $k = 1, 2$ cases only.

JHEP08(2009)093

3 Supersymmetry enhancement of the ABJM theory with U(2)×U(2) gauge group

The ABJM action with U(N)×U(N) gauge group at Chern-Simons level (k,$-k$) in SU(4) invariant form is given by

$$S = \int d^3x \; (\mathcal{L}_0 + \mathcal{L}_{\rm CS} - V_{\rm ferm} - V_{\rm bos}) \tag{3.1}$$

with

$$\mathcal{L}_0 = \mathrm{tr}\left(-D_\mu Y_A^\dagger D^\mu Y^A + i\psi^{\dagger A}\gamma^\mu D_\mu \psi_A\right), \tag{3.2}$$

$$\mathcal{L}_{\rm CS} = \frac{k}{4\pi}\,\epsilon^{\mu\nu\rho}\,\mathrm{tr}\left(A_\mu\partial_\nu A_\rho + \frac{2i}{3}A_\mu A_\nu A_\rho - \hat{A}_\mu\partial_\nu\hat{A}_\rho - \frac{2i}{3}\hat{A}_\mu\hat{A}_\nu\hat{A}_\rho\right), \tag{3.3}$$

$$\begin{aligned} V_{\rm ferm} = \frac{2\pi i}{k}\mathrm{tr}\Big(& Y_A^\dagger Y^A\psi^{\dagger B}\psi_B - Y^A Y_A^\dagger \psi_B\psi^{\dagger B} + 2Y^A Y_B^\dagger \psi_A \psi^{\dagger B} - 2Y_A^\dagger Y^B\psi^{\dagger A}\psi_B \\ &+ \epsilon^{ABCD}Y_A^\dagger\psi_B Y_C^\dagger\psi_D - \epsilon_{ABCD}Y^A\psi^{\dagger B}Y^C\psi^{\dagger D}\Big), \end{aligned} \tag{3.4}$$

$$\begin{aligned} V_{\rm bos} = -\frac{4\pi^2}{3k^2}\mathrm{tr}\Big(& Y_A^\dagger Y^A Y_B^\dagger Y^B Y_C^\dagger Y^C + Y^A Y_A^\dagger Y^B Y_B^\dagger Y^C Y_C^\dagger + 4Y_A^\dagger Y^B Y_C^\dagger Y^A Y_B^\dagger Y^C \\ &- 6Y^A Y_B^\dagger Y^B Y_A^\dagger Y^C Y_C^\dagger\Big). \end{aligned} \tag{3.5}$$

Here the complex scalars Y^A, ($A = 1,\cdots,4$), the fermions ψ_A are in bifundamental representation, and the covariant derivatives are same as (2.6).

The ABJM action (3.1) is invariant under $\mathcal{N} = 6$ supersymmetry transformation,

$$\begin{aligned} \delta Y^A &= i\omega^{AB}\psi_B, \\ \delta Y_A^\dagger &= i\psi^{\dagger B}\omega_{AB}, \\ \delta\psi_A &= \gamma^\mu\omega_{AB}D_\mu Y^B + \frac{2\pi}{k}\omega_{AB}(Y^B Y_C^\dagger Y^C - Y^C Y_C^\dagger Y^B) + \frac{4\pi}{k}\omega_{BC}Y^B Y_A^\dagger Y^C, \\ \delta\psi^{\dagger\,A} &= -D_\mu Y_B^\dagger \omega^{AB}\gamma^\mu + \frac{2\pi}{k}\omega^{AB}(Y_C^\dagger Y^C Y_B^\dagger - Y_B^\dagger Y^C Y_C^\dagger) - \frac{4\pi}{k}\omega^{BC}Y_B^\dagger Y^A Y_C^\dagger, \\ \delta A_\mu &= -\frac{2\pi}{k}(\omega^{AB}Y_A^\dagger\gamma_\mu\psi_B + Y^A\psi^{\dagger B}\gamma_\mu\omega_{AB}), \\ \delta\hat{A}_\mu &= -\frac{2\pi}{k}(\omega^{AB}Y_A^\dagger\gamma_\mu\psi_B + \psi^{\dagger B}\gamma_\mu Y^A\omega_{AB}), \end{aligned} \tag{3.6}$$

where $\omega^{AB} = -\omega^{BA} = (\omega_{AB})^* = \frac{1}{2}\,\epsilon^{ABCD}\omega_{CD}$.

3.1 U(2)×U(2) case

It was conjectured that the ABJM theory with U(N)×U(N) gauge group has the additional $\mathcal{N} = 2$ supersymmetry at $k = 1, 2$, in addition to $\mathcal{N} = 6$ supersymmetry written in (3.6) [4]. Since the equivalence between the SU(2)×SU(2) ABJM theory and BLG theory which has $\mathcal{N} = 8$ supersymmetry was known in ref. [4] already, we know that the ABJM theory with SU(2)×SU(2) gauge group has $\mathcal{N} = 8$ supersymmetry without dependence of k, i.e. there is $\mathcal{N} = 2$ supersymmetry enhancement in SU(2)×SU(2) case though the supersymmetry transformation rules for component fields were not known up to now. In appendix B.1, we verify the supersymmetry invariance of the ABJM theory with SU(2)×SU(2) gauge group and find the corresponding additional $\mathcal{N}=2$ supersymmetry transformation rules. On the other hand, for U(1)×U(1) case, the $\mathcal{N}=2$ minimal model and ABJM theory have the same kinetic and Chern-Simons terms. So they have the same supersymmetric behaviors for the additional $\mathcal{N}=2$ supersymmetry, though the numbers of matter fields are different. Actually the additional $\mathcal{N}=2$ supersymmetry given in (2.17) for U(1)×U(1) case does not dependent on the number of matter fields.

As we discussed in the subsection 2.3, in order to obtain the additional supersymmetry transformation rules of U(2)×U(2) we can combine the results of U(1)×U(1) and SU(2)×SU(2) cases. The results are as follows:

$$\begin{aligned}
\delta Y^A &= i\check{T}\tilde{\psi}^{\dagger A}\tilde{\varepsilon}, \\
\delta Y^\dagger_A &= i\tilde{\varepsilon}^\dagger \check{T}^* \tilde{\psi}_A, \\
\delta\psi_A &= -\check{T}D_\mu \tilde{Y}^\dagger_A \gamma^\mu \tilde{\varepsilon} - \check{T}\tilde{N}^\dagger_A \tilde{\varepsilon} - \frac{4\pi}{3k}\check{T}^*\tilde{\varepsilon}^\dagger \epsilon_{ABCD} Y^B \tilde{Y}^C Y^D, \\
\delta\psi^{\dagger A} &= \check{T}^*\tilde{\varepsilon}^\dagger \gamma^\mu D_\mu \tilde{Y}^A - \check{T}^*\tilde{\varepsilon}^\dagger \tilde{N}^A + \frac{4\pi}{3k}\check{T}\epsilon^{ABCD} Y^\dagger_B \tilde{Y}^\dagger_C Y^\dagger_D \tilde{\varepsilon}, \\
\delta A_\mu &= \frac{2\pi}{k}\left(\check{T}^*\tilde{\varepsilon}^\dagger Y^A \gamma_\mu \tilde{\psi}_A + \check{T}\tilde{\psi}^{\dagger A}\gamma_\mu Y^\dagger_A \tilde{\varepsilon}\right), \\
\delta \hat{A}_\mu &= \frac{2\pi}{k}\left(\check{T}^*\tilde{\varepsilon}^\dagger \gamma_\mu \tilde{\psi}_A Y^A + \check{T} Y^\dagger_A \tilde{\psi}^{\dagger A}\gamma_\mu \tilde{\varepsilon}\right),
\end{aligned} \tag{3.7}$$

where $\tilde{Y}^A$, $\tilde{\psi}_A$, and $\tilde{N}^A$ are defined in (2.25) by replacing Z with Y.

Similarly to the case of the $\mathcal{N}=2$ minimal model with U(2)×U(2) gauge group discussed in the subsection 2.3, the supersymmetry of the $\mathcal{N}=6$ ABJM theory with U(2)×U(2) gauge group is enhanced to $\mathcal{N}=8$ for $k=1,2$ cases due to the presence of the abelian monopole operator $\check{T}$.

3.2 Comments on U(N)×U(N) case

Now we try to extend our results to SU(N)×SU(N) case[3]. Here, we only give a sketchy of the whole procedure whose explicit construction for the supersymmetry transformation rules will complete of the supersymmetry enhancement in ABJM theory. The details will be published elsewhere.

As we did in appendix to prove the supersymmetry invariance of the given Lagrangian in Chern-Simons matter theory, we start from the following variations for the scalars and fermions:

$$\begin{aligned}
\delta_1 Y^{\dagger\hat{a}}_{A\ a} &= i\tilde{\varepsilon}^\dagger T^{\hat{a}\hat{b}}_{ab}\psi^b_{A\hat{b}}, \\
\delta_1 \psi^{\dagger A\hat{a}}_{\ \ a} &= \tilde{\varepsilon}^\dagger T^{\hat{a}\hat{b}}_{ab}\gamma^\mu D_\mu Y^{Ab}_{\ \ \hat{b}},
\end{aligned} \tag{3.8}$$

[3]Except for U(1)×U(1) factor, there is no difference between U(N)×U(N) and SU(N)×SU(N) cases in obtaining supersymmetry transformation rules. As we have seen in the subsection 2.3, we can consider the U(1)×U(1) part separately. So concentrating on SU(N)×SU(N) case can cover the U(N)×U(N) case also.

JHEP08(2009)093

where we denoted the gauge indices for concreteness. Since all the gauge groups except for SU(2)×SU(2) include the U(1)×U(1) part, the local operator T transforms as $T \to e^{-4\pi i/k}T$ under the orbifold transformation for the generic gauge group. Therefore, the supersymmetry transformation (3.8) is satisfied in the cases $k = 1, 2$ only. Here $D_\mu Y^{Aa}{}_{\hat{a}}$, $(a, b, \ldots = 1, 2, \ldots N; \hat{a}, \hat{b}, \ldots = 1, 2, \ldots N)$, and its Hermitian conjugate are given by

$$
\begin{aligned}
D_\mu Y^{Aa}{}_{\hat{a}} &= \partial_\mu Y^{Aa}{}_{\hat{a}} + iA^a_{\mu b} Y^{Ab}{}_{\hat{a}} - iY^{Aa}{}_{\hat{b}} \hat{A}^{\hat{b}}_{\mu\hat{a}}, \\
D_\mu Y^{\dagger\hat{a}}_{A\ a} &= \partial_\mu Y^{\dagger\hat{a}}_{A\ a} + i\hat{A}^{\hat{a}}_{\mu\hat{b}} Y^{\dagger\hat{b}}_{A\ a} - iY^{\dagger\hat{a}}_{A\ b} A^b_{\mu a}.
\end{aligned} \tag{3.9}
$$

The variation of $\mathcal{L}_0$ in (3.2) in matrix notation is given by

$$
\begin{aligned}
\delta_1\mathcal{L}_0 &= \mathrm{tr}\Big(- D_\mu \delta_1 Y^\dagger_A D^\mu Y^A + i\delta_1\psi^{\dagger A}\gamma^\mu D_\mu \psi_A \Big) \\
&= \mathrm{tr}\Big(- i\tilde{\varepsilon}^\dagger D_\mu(T\psi_A) D^\mu Y^A + i\tilde{\varepsilon}^\dagger T D_\mu Y^A \gamma^\mu\gamma^\nu D_\nu\psi_A \Big) \\
&= \mathrm{tr}\Big(- i\tilde{\varepsilon}^\dagger \gamma^\nu\gamma^\mu D_\mu T\psi_A D_\nu Y^A - \frac{1}{2}\tilde{\varepsilon}^\dagger\epsilon^{\mu\nu\rho} T F_{\mu\nu} Y^A \gamma_\rho\psi_A + \frac{1}{2}\tilde{\varepsilon}^\dagger\epsilon^{\mu\nu\rho} T Y^A \hat{F}_{\mu\nu}\gamma_\rho\psi_A \Big).
\end{aligned} \tag{3.10}
$$

In the final step of (3.10) we integrated by part, dropped the total derivative term, and used the following relation,

$$
D_\mu(T\psi_A) = (D_\mu T)\psi_A + T D_\mu\psi_A. \tag{3.11}
$$

Denoting the gauge indices we can express the left hand side of (3.11) as

$$
D_\mu(T\psi_A)^{\hat{a}}{}_a = (\partial_\mu T^{\hat{a}\hat{b}}_{ab})\psi^b_{A\hat{b}} + T^{\hat{a}\hat{b}}_{ab}(\partial_\mu\psi^b_{A\hat{b}}) + i\hat{A}^{\hat{a}}_{\mu\hat{b}} T^{\hat{b}\hat{c}}_{ac}\psi^c_{A\hat{c}} - iT^{\hat{a}\hat{c}}_{bc}\psi^c_{A\hat{c}} A^b_{\mu a} \tag{3.12}
$$

and the right hand side of (3.11) as

$$
((D_\mu T)\psi_A)^{\hat{a}}{}_a + (TD_\mu\psi_A)^{\hat{a}}{}_a = (D_\mu T)^{\hat{a}\hat{b}}_{ab}\psi^b_{A\hat{b}} + T^{\hat{a}\hat{b}}_{ab}\left(\partial_\mu\psi^b_{A\hat{b}} + iA^b_{\mu c}\psi^c_{A\hat{b}} - i\psi^b_{A\hat{c}}\hat{A}^{\hat{c}}_{\mu\hat{b}}\right). \tag{3.13}
$$

Combining (3.11), (3.12), and (3.13), we obtain

$$
(D_\mu T)^{\hat{a}\hat{b}}_{ab}\psi^b_{A\hat{b}} = \left[\partial_\mu T^{\hat{a}\hat{b}}_{ab} + i\hat{A}^{\hat{a}}_{\mu\hat{c}} T^{\hat{c}\hat{b}}_{ab} + i\hat{A}^{\hat{b}}_{\mu\hat{c}} T^{\hat{a}\hat{c}}_{ab} - iT^{\hat{a}\hat{b}}_{cb} A^c_{\mu a} - iT^{\hat{a}\hat{b}}_{ac} A^c_{\mu b}\right]\psi^b_{A\hat{b}}. \tag{3.14}
$$

On the other hand, the variation of $\mathcal{L}_{\mathrm{CS}}$ in (3.3) is given by

$$
\delta_A\mathcal{L}_{\mathrm{CS}} = \frac{k}{4\pi}\epsilon^{\mu\nu\rho}\mathrm{tr}\left(F_{\mu\nu}\delta A_\rho - \delta\hat{A}_\rho\hat{F}_{\mu\nu}\right). \tag{3.15}
$$

Adding (3.10) and (3.15) we obtain

$$
\begin{aligned}
\delta_1\mathcal{L}_0 + \delta_A\mathcal{L}_{\mathrm{CS}} = &- i\tilde{\varepsilon}^\dagger\gamma^\nu\gamma^\mu D_\mu T^{\hat{a}\hat{b}}_{ab}\psi^b_{A\hat{b}} D_\nu Y^{Aa}{}_{\hat{a}} \\
&- \frac{1}{2}\tilde{\varepsilon}^\dagger\epsilon^{\mu\nu\rho} T^{\hat{a}\hat{b}}_{ab} F^b_{\mu\nu c} Y^{Ac}{}_{\hat{b}}\gamma_\rho\psi^a_{A\hat{a}} + \frac{1}{2}\tilde{\varepsilon}^\dagger\epsilon^{\mu\nu\rho} T^{\hat{a}\hat{b}}_{ab} Y^{Ab}{}_{\hat{c}}\hat{F}^{\hat{c}}_{\mu\nu\hat{b}}\gamma_\rho\psi^a_{A\hat{a}} \\
&+ \frac{k}{4\pi}\epsilon^{\mu\nu\rho} F^b_{\mu\nu c}\delta A^c_{\rho b} - \frac{k}{4\pi}\epsilon^{\mu\nu\rho}\delta\hat{A}^{\hat{b}}_{\rho\hat{c}}\hat{F}^{\hat{c}}_{\mu\nu\hat{b}}.
\end{aligned} \tag{3.16}
$$

As we did in the relations (A.4) and (A.16) which appeared in proving the supersymmetry invariance of the Chern-Simons matter theories, we first impose the vanishing of the right hand side of (3.16). Then we obtain the variations for the gauge field and a condition for $T_{ab}^{\hat{a}\hat{b}}$ as follows:

$$\begin{aligned}
\delta A_{\mu b}^{a} &= \frac{2\pi}{k}\tilde{\varepsilon}^{\dagger}Y^{Aa}{}_{\hat{a}}\gamma_{\mu}T_{bc}^{\hat{a}\hat{b}}\psi_{A\hat{b}}^{c}, \\
\delta \hat{A}_{\mu\hat{b}}^{\hat{a}} &= \frac{2\pi}{k}\tilde{\varepsilon}^{\dagger}\gamma_{\mu}T_{ab}^{\hat{a}\hat{c}}\psi_{A\hat{c}}^{b}Y^{Aa}{}_{\hat{b}},
\end{aligned} \tag{3.17}$$

$$(D_{\mu}T)_{ab}^{\hat{a}\hat{b}} = \partial_{\mu}T_{ab}^{\hat{a}\hat{b}} + i\hat{A}_{\mu\hat{c}}^{\hat{a}}T_{ab}^{\hat{c}\hat{b}} + i\hat{A}_{\mu\hat{c}}^{\hat{b}}T_{ab}^{\hat{a}\hat{c}} - iT_{cb}^{\hat{a}\hat{b}}A_{\mu a}^{c} - iT_{ac}^{\hat{a}\hat{b}}A_{\mu b}^{c} = 0. \tag{3.18}$$

The supersymmetry transformation rules for SU(N)×SU(N) case are read from (3.8) and (3.17). Though Y^3-terms in the variation of fermion fields, which are originated from the variation of the potentials are not available, the rules (3.8) and (3.17) will be very useful in finding the complete supersymmetry transformation rules in SU(N)×SU(N) case.

4 Conclusion

We investigated the supersymmetry enhancement behaviors of the ABJM theory. We found the additional $\mathcal{N}=2$ supersymmetry explicitly for U(1)×U(1) and U(2)×U(2) cases at $k = 1, 2$, by introducing the local operator $\check{T}$ which is known as monopole operator. In obtaining the additional supersymmetry transformation rules, we started from the verification of the supersymmetric invariance for the minimal model which has the same supersymmetry enhancement properties as those of ABJM theory. The minimal model is a $\mathcal{N}=2$ supersymmetric Chern-Simons matter theory with U(1)$_R$ symmetry and has the same forms of the kinetic terms for scalars and fermions, the Chern-Simons terms. The matter field part is composed of two complex scalars and fermions. We found the explicit supersymmetry transformation rules for $\mathcal{N}=2$ supersymmetry coming from the global symmetry and the additional $\mathcal{N}=2$ supersymmetry originated from the gauge part for U(1)×U(1) and U(2)×U(2) at $k = 1, 2$. That is, the minimal model has $\mathcal{N}=4$ supersymmetry at $k = 1, 2$.

The procedure of the minimal model can be repeated to ABJM theory, and we proved the conjecture for the supersymmetry enhancement for U(1)×U(1) and U(2)×U(2) cases. We explicitly obtained the additional $\mathcal{N}=2$ supersymmetry transformation rules by using the monopole operator and showed $\mathcal{N}=8$ supersymmetry of ABJM theory. We also studied the additional $\mathcal{N}=2$ supersymmetry for the generic gauge group U(N)×U(N) case without the contribution from the potentials of ABJM theory, and derived a condition for the monopole operator T, which satisfies first order coupled differential equation. Since T has four indices, solving the coupled differential equations is nontrivial and we were not able to complete the computation of Y^3-terms in the variation of the fermion fields, which seems to require a more lengthy calculation. But we believe that the variations for the potential part give some additional condition for T and we can reduce the degrees of freedom for T considerably.

JHEP08(2009)093

As the extensions of the works related to the $\mathcal{N}=6$ supersymmetry of ABJM theory, there are several directions we can consider by using the additional $\mathcal{N}=2$ supersymmetry (3.7), for instance, supersymmetry preserving mass deformation [7, 23–25], various soliton solutions [24, 26–31].

We conclude with a final remark. Recently the non-relativistic limit of ABJM theory was obtained in refs. [32, 33]. In the non-relativistic theories the $\mathcal{N}=6$ part of supersymmetry of ABJM theory was reduced to the kinematical, dynamical, and conformal charges. It is also interesting to consider the non-relativistic reduction for the additional $\mathcal{N}=2$ part of supersymmetry of ABJM theory [34].

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A Supersymmetry of the $\mathcal{N}=2$ minimal model

A.1 Verification of (2.7)

Let us check the $\mathcal{N}=2$ supersymmetry (2.7) of the action (2.1). As we did in the U(1)×U(1) case in the subsection 2.1, we only check half of terms. The remaining half are complex conjugate of them. From now on, we drop total derivatives in all calculational procedures.

A supersymmetric variation of the kinetic term $\mathcal{L}_0$ (2.2) is given by

$$\begin{aligned}\delta_1\mathcal{L}_0 &= \mathrm{tr}\left(-D_\mu Z_A^\dagger D^\mu \delta_1 Z^A + i\delta_1\psi^{\dagger B}\gamma^\nu D_\nu\psi_B\right)\\ &= \mathrm{tr}\left[\frac{1}{2}\epsilon^{\mu\nu\rho}F_{\mu\nu}\left(\varepsilon^\dagger\epsilon^{AB}\gamma_\rho\psi_B Z_A^\dagger\right) - \frac{1}{2}\epsilon^{\mu\nu\rho}\left(\varepsilon^\dagger\epsilon^{AB}Z_A^\dagger\gamma_\rho\psi_B\right)\hat{F}_{\mu\nu}\right],\end{aligned}\tag{A.1}$$

where $F_{\mu\nu}$ and $\hat{F}_{\mu\nu}$ are gauge field strengths of A_μ and $\hat{A}_\mu$ respectively, and

$$\begin{aligned}\delta_1 Z^A &= i\varepsilon^\dagger\epsilon^{AB}\psi_B,\\ \delta_1\psi^{\dagger B} &= \varepsilon^\dagger\epsilon^{AB}D_\mu Z_A^\dagger\gamma^\mu.\end{aligned}\tag{A.2}$$

In the last step of (A.1), we used integration by part and

$$[D_\mu,\, D_\nu]Z_A^\dagger = i\hat{F}_{\mu\nu}Z_A^\dagger - iZ_A^\dagger F_{\mu\nu}.$$

We add $\delta_1\mathcal{L}_0$ in (A.1) to supersymmetric variations of gauge fields in the Chern-Simons terms,

$$\delta_A\mathcal{L}_{\mathrm{CS}} = \mathrm{tr}\left(\frac{k}{4\pi}\epsilon^{\mu\nu\rho}F_{\mu\nu}\delta A_\rho - \frac{k}{4\pi}\epsilon^{\mu\nu\rho}\delta\hat{A}_\rho\hat{F}_{\mu\nu}\right).$$

JHEP08(2009)093

If we set the supersymmetric variations of gauge fields as follows,

$$\delta A_\mu = -\frac{2\pi}{k}\varepsilon^\dagger \epsilon^{AB}\gamma_\mu \psi_B Z_A^\dagger,$$
$$\delta \hat{A}_\mu = -\frac{2\pi}{k}\varepsilon^\dagger \epsilon^{AB} Z_A^\dagger \gamma_\mu \psi_B, \tag{A.3}$$

we obtain

$$\delta_1 \mathcal{L}_0 + \delta_A \mathcal{L}_{\rm CS} = 0. \tag{A.4}$$

Now we consider the variation of the kinetic part $\mathcal{L}_0$ originated form the variations of gauge fields,

$$\delta_A \mathcal{L}_0 = \mathrm{tr}\Big[iZ_C^\dagger \delta A^\mu D_\mu Z^C - iD_\mu Z_C^\dagger \delta A^\mu Z^C - \psi^{\dagger C}\gamma^\mu \delta A_\mu \psi_C$$
$$-i\delta\hat{A}^\mu Z_C^\dagger D_\mu Z^C + iD_\mu Z_C^\dagger Z^C \delta\hat{A}^\mu + \psi^{\dagger C}\gamma^\mu \psi_C \delta\hat{A}_\mu \Big]. \tag{A.5}$$

Plugging (A.3) into (A.5), we obtain, after some algebra, the following relation,

$$\delta_A \mathcal{L}_0 = -\mathrm{tr}\left(i\delta_2 \psi^{\dagger B}\gamma^\mu D_\mu \psi_B \right) + \delta_1 V_{\rm ferm}$$
$$= -\delta_2 \mathcal{L}_0 + \delta_1 V_{\rm ferm}, \tag{A.6}$$

where

$$\delta_2 \psi^{\dagger B} = -\frac{2\pi}{k}\varepsilon^\dagger \epsilon^{AB}\left(Z_A^\dagger Z^C Z_C^\dagger - Z_C^\dagger Z^C Z_A^\dagger \right), \tag{A.7}$$

$$V_{\rm ferm} = \frac{2\pi i}{k}\mathrm{tr}\left(Z_A^\dagger Z^A \psi^{\dagger B}\psi_B - Z^A Z_A^\dagger \psi_B \psi^{\dagger B} + 2Z^A Z_B^\dagger \psi_A \psi^{\dagger B} - 2Z_A^\dagger Z^B \psi^{\dagger A}\psi_B \right). \tag{A.8}$$

As a next step, we consider the variation of $V_{\rm ferm}$ originated from $\delta_2\psi^{\dagger B}$ given in (A.7),

$$\delta_2 V_{\rm ferm} = \frac{2\pi i}{k}\mathrm{tr}\Big(\delta_2\psi^{\dagger B}\psi_B Z_D^\dagger Z^D - \delta_2\psi^{\dagger B} Z^D Z_D^\dagger \psi_B$$
$$+ 2\delta_2\psi^{\dagger B} Z^D Z_B^\dagger \psi_D - 2\delta_2\psi^{\dagger B}\psi_D Z_B^\dagger Z^D \Big)$$
$$= -\delta_1 V_{\rm bos}, \tag{A.9}$$

where

$$V_{\rm bos} = \frac{4\pi}{k^2}\mathrm{tr}\left(Z_A^\dagger Z^A Z_B^\dagger Z^B Z_C^\dagger Z^C + Z^A Z_A^\dagger Z^B Z_B^\dagger Z^C Z_C^\dagger - 2Z^A Z_B^\dagger Z^B Z_A^\dagger Z^C Z_C^\dagger \right). \tag{A.10}$$

Since there is no fermion field in the expression of $V_{\rm bos}$, we see that $\delta_1 V_{\rm bos} = (\delta_1+\delta_2)V_{\rm bos} = \delta V_{\rm bos}$.

From the relations, (A.2), (A.3), and (A.7), we obtain the total supersymmetric variations for component fields,

$$\delta Z^A = \delta_1 Z^A,$$
$$\delta\psi^{\dagger A} = \delta_1\psi^{\dagger A} + \delta_2\psi^{\dagger A}.$$

Then the total supersymmetric variation for the Lagrangian in (2.1) can be obtained from the equations, (A.4), (A.6), and (A.9), as follows,

$$\delta(\mathcal{L}_0 + \mathcal{L}_{\rm CS} - V_{\rm ferm} - V_{\rm bos}) = \delta\mathcal{L} = 0.$$

Adding the complex conjugate parts of the supersymmetry transformation rules, we prove that the action (2.1) is invariant under the $\mathcal{N}=2$ supersymmetry given in (2.7).

JHEP08(2009)093

A.2 Verification of (2.24)

As we did in the previous subsection, we start form a variation of the kinetic part $\mathcal{L}_0$ given in (2.2),

$$\begin{aligned}\delta_1\mathcal{L}_0 &= \mathrm{tr}\left(-D_\mu\delta_1 Z_A^\dagger D^\mu Z^A + i\delta_1\psi^{\dagger A}\gamma^\nu D_\nu\psi_A\right)\\ &= \mathrm{tr}\left(-\frac{1}{2}\epsilon^{\mu\nu\rho}F_{\mu\nu}(\tilde\varepsilon^\dagger Y^A\gamma_\rho\tilde\psi_A) + \frac{1}{2}\epsilon^{\mu\nu\rho}(\tilde\varepsilon^\dagger\gamma_\rho\tilde\psi_A Y^A)\hat F_{\mu\nu}\right),\end{aligned} \tag{A.11}$$

where $\tilde Y^A$ and $\tilde\psi_A$ were defined in (2.25) and

$$\begin{aligned}\delta_1 Z_A^\dagger &= i\tilde\varepsilon^\dagger\tilde\psi_A,\\ \delta_1\psi^{\dagger A} &= D_\mu\tilde Z^A\tilde\varepsilon^\dagger\gamma^\mu.\end{aligned} \tag{A.12}$$

In the last step of (A.11), we used the properties of SU(2) invariant tensors, ϵ^{ab} and $\epsilon^{\hat a\hat b}$ with gauge group indices $a,b=1,2$, for instance,

$$\begin{aligned}D_\mu\tilde Z^{A\hat a}{}_a &= \partial_\mu\tilde Z^{\hat a}{}_a + i\hat A^{\hat a}_{\mu\hat b}\tilde Z^{A\hat b}{}_a - i\tilde Z^{A\hat a}{}_b A^b_{\mu a}\\ &= \epsilon^{\hat a\hat b}\epsilon_{ab}\left(\partial_\mu Z^{Ab}{}_{\hat b} + iA^b_{\mu c}Z^{Ac}{}_{\hat b} - iZ^{Ab}{}_{\hat c}\hat A^{\hat c}_{\mu\hat b}\right)\\ &= \epsilon^{\hat a\hat b}\epsilon_{ab}(D_\mu Z^A)^b{}_{\hat b}\end{aligned} \tag{A.13}$$

with the help of symmetric properties,

$$\begin{aligned}(\epsilon A_\mu)_{ac} &= \epsilon_{ab}A^b_{\mu c} = \epsilon_{cb}A^b_{\mu a} = (\epsilon A_\mu)_{ca},\\ (\hat A_\mu\epsilon)^{\hat a\hat c} &= \hat A^{\hat a}_{\mu\hat b}\epsilon^{\hat b\hat c} = \hat A^{\hat c}_{\mu\hat b}\epsilon^{\hat b\hat a} = (\hat A_\mu\epsilon)^{\hat c\hat a}.\end{aligned} \tag{A.14}$$

By taking supersymmetry variations for the gauge fields, A_μ and $\hat A_\mu$, as

$$\begin{aligned}\delta A_\mu &= \frac{2\pi}{k}\tilde\varepsilon^\dagger Z^A\gamma_\mu\tilde\psi_A,\\ \delta\hat A_\mu &= \frac{2\pi}{k}\tilde\varepsilon^\dagger\gamma_\mu\tilde\psi_A Z^A,\end{aligned} \tag{A.15}$$

we obtain the same relation given in (A.4), i.e.,

$$\delta_1\mathcal{L}_0 + \delta_A\mathcal{L}_{\mathrm{CS}} = 0. \tag{A.16}$$

Now we consider the variations of the gauge fields in $\mathcal{L}_0$, and obtain the following relation

$$\begin{aligned}\delta_A\mathcal{L}_0 &= \mathrm{tr}\Big(-i\delta_2\psi^{\dagger A}\gamma^\mu D_\mu\psi_A\Big) + \delta_1 V_{\mathrm{ferm}}\\ &= -\delta_2\mathcal{L}_0 + \delta_1 V_{\mathrm{ferm}},\end{aligned} \tag{A.17}$$

where V_{ferm} was given in (A.8) and

$$\delta_2\psi^{\dagger A} = -\tilde\varepsilon^\dagger\tilde N^A \tag{A.18}$$

with the definition of $\tilde A^A$ in (2.25).

From the variation $\delta_2\psi^{\dagger A}$ in $V_{\rm ferm}$, we can obtain the bosonic potential $V_{\rm bos}$,

$$\delta_2 V_{\rm ferm} = -\delta_1 V_{\rm bos} = -\delta V_{\rm bos}, \tag{A.19}$$

where the expression of $V_{\rm bos}$ was given in (A.10). During the calculational procedures in obtaining (A.17) and (A.19), we frequently used the following relations

$$\begin{aligned}
\epsilon_{ab}P^aQ^bR^c &= \epsilon_{ab}(-P^cQ^a + P^aQ^c)R^b,\\
(\tilde\varepsilon^\dagger\psi_1)(\psi_2\psi_3) &= -(\tilde\varepsilon^\dagger\psi_2)(\psi_3\psi_1) - (\tilde\varepsilon^\dagger\psi_3)(\psi_2\psi_1),\\
(\tilde\varepsilon^\dagger\gamma_\mu\psi_1)(\psi_2\gamma^\mu\psi_3) &= -2(\tilde\varepsilon^\dagger\psi_3)(\psi_1\psi_2) - (\tilde\varepsilon^\dagger\psi_1)(\psi_2\psi_3),
\end{aligned} \tag{A.20}$$

where P, Q, and R are arbitrary fields with gauge indices and ψ_1, ψ_2, and ψ_3 are arbitrary fermion fields.

Combining the relations (A.16), (A.17), and (A.19), as we did in the previous subsection, we can verify the supersymmetry transformation rules given in (2.24).

B Supersymmetry of ABJM theory

B.1 SU(2)×SU(2) case

If we replace Z with Y and extend the SU(2) global indices $A, B, \ldots = 1, 2$ to the SU(4) global indices $A, B, \ldots = 1, 2, 3, 4$ in (2.1), the kinetic and Chern-Simons terms are exactly same with those in (3.1), while the fermionic and bosonic potentials have different forms in the two actions. In this reason, in order to verify the additional $\mathcal{N} = 2$ supersymmetry invariance given in (3.7) (without $\check T$) in ABJM theory with SU(2)×SU(2) gauge group, we use the results given in the subsection A.2. The results in the subsection A.2 are summarized in ABJM theory side, as follows,

$$\begin{aligned}
\delta_1\mathcal{L}_0 + \delta_A\mathcal{L}_{\rm CS} &= 0,\\
\delta_A\mathcal{L}_0 + \delta_2\mathcal{L}_0 - \delta_1 V_{\rm ferm1} &= 0,\\
\delta_2 V_{\rm ferm1} + \delta_1 V_{\rm bos1} &= 0,
\end{aligned} \tag{B.1}$$

where

$$\begin{aligned}
\delta_1 Y_A^\dagger &= i\tilde\varepsilon^\dagger\tilde\psi_A,\\
\delta_1\psi^{\dagger A} &= D_\mu\tilde Y^A\tilde\varepsilon^\dagger\gamma^\mu,\\
\delta_2\psi^{\dagger A} &= -\tilde\varepsilon^\dagger\tilde N^A,\\
\delta A_\mu &= \frac{2\pi}{k}\tilde\varepsilon^\dagger Y^A\gamma_\mu\tilde\psi_A,\\
\delta\hat A_\mu &= \frac{2\pi}{k}\tilde\varepsilon^\dagger\gamma_\mu\tilde\psi_A Y^A
\end{aligned} \tag{B.2}$$

with $\tilde N^{A\hat a}{}_a = \epsilon^{\hat a\hat b}\epsilon_{ab}\frac{2\pi}{k}(Y^BY_B^\dagger Y^A - Y^AY_B^\dagger Y^B)^b{}_{\hat b}$, and

$$V_{\rm ferm1} = \frac{2\pi i}{k}{\rm tr}\left(Y_A^\dagger Y^A\psi^{\dagger B}\psi_B - Y^AY_A^\dagger\psi_B\psi^{\dagger B} + 2Y^AY_B^\dagger\psi_A\psi^{\dagger B} - 2Y_A^\dagger Y^B\psi^{\dagger A}\psi_B\right), \tag{B.3}$$

$$V_{\rm bos1} = \frac{4\pi^2}{k^2}{\rm tr}\left(Y_A^\dagger Y^AY_B^\dagger Y^BY_C^\dagger Y^C + Y^AY_A^\dagger Y^BY_B^\dagger Y^CY_C^\dagger - 2Y^AY_B^\dagger Y^BY_A^\dagger Y^CY_C^\dagger\right). \tag{B.4}$$

In the next, we consider a supersymmetry variation of V_{ferm} defined as

$$\begin{aligned} V_{\text{ferm2}} &= V_{\text{ferm}} - V_{\text{ferm1}} \\ &= \frac{2\pi i}{k}\text{tr}\left(\epsilon^{ABCD}Y_A^\dagger\psi_B Y_C^\dagger\psi_D - \epsilon_{ABCD}Y^A\psi^{\dagger B}Y^C\psi^{\dagger D}\right), \end{aligned} \tag{B.5}$$

where V_{ferm} and V_{ferm1} were defined in (3.4) and (B.3) respectively.

Then we find

$$\begin{aligned} \delta_1 V_{\text{ferm2}} &= \frac{4\pi i}{k}\text{tr}\left(\epsilon^{ABCD}\delta_1 Y_A^\dagger\psi_B Y_C^\dagger\psi_D - \epsilon_{ABCD}\delta_1\psi^{\dagger A}Y^B\psi^{\dagger C}Y^D\right) \\ &= i\delta_3\psi_A\gamma^\mu D_\mu\psi^{\dagger A} \\ &= \partial_\mu(i\delta_3\psi_A\gamma^\mu\psi^{\dagger A}) + i\psi^{\dagger A}\gamma^\mu D_\mu\delta_3\psi_A, \end{aligned} \tag{B.6}$$

where

$$\delta_3\psi_A = -\frac{4\pi}{3k}\epsilon_{ABCD}\tilde{\varepsilon}^\dagger Y^B\tilde{Y}^C Y^D. \tag{B.7}$$

Here we used the facts that

$$\begin{aligned} &\text{tr}\left(\epsilon^{ABCD}\delta_1 Y_A^\dagger\psi_B Y_C^\dagger\psi_D\right) = 0, \\ &\text{tr}\left(\epsilon_{ABCD}D_\mu\tilde{Y}^A\tilde{\varepsilon}^\dagger\gamma^\mu Y^B\psi^{\dagger C}Y^D\right) = -\frac{1}{3}\text{tr}\left(\epsilon_{ABCD}Y^B\tilde{Y}^C Y^D\tilde{\varepsilon}^\dagger\gamma^\mu D_\mu\psi^{\dagger A}\right). \end{aligned} \tag{B.8}$$

After slightly long algebra, we obtain

$$\delta_2 V_{\text{ferm2}} = -\delta_3 V_{\text{ferm1}}, \tag{B.9}$$

where

$$\begin{aligned} \delta_2 V_{\text{ferm2}} &= \frac{4\pi i}{k}\text{tr}\left(\epsilon_{ABCD}\delta_2\psi^{\dagger A}Y^B\psi^{\dagger C}Y^D\right), \\ \delta_3 V_{\text{ferm1}} &= \frac{2\pi i}{k}\text{tr}\Big(\delta_3\psi_A Y_B^\dagger Y^B\psi^{\dagger A} - \delta_3\psi_A\psi^{\dagger A}Y^B Y_B^\dagger \\ &\qquad + 2\delta_3\psi_A\psi^{\dagger B}Y^A Y_B^\dagger - 2\delta_3\psi_A Y_B^\dagger Y^A\psi^{\dagger B}\Big). \end{aligned}$$

As a final step we check $\delta_3 V_{\text{ferm2}}$ and find the corresponding bosonic potential. By expending $\epsilon^{ABCD}\epsilon_{AEFG}$ and using the properties of the SU(2) invariant tensors, we finally find the following relation

$$\delta_3 V_{\text{ferm2}} = -\delta_1 V_{\text{bos2}}, \tag{B.10}$$

where

$$\begin{aligned} \delta_3 V_{\text{ferm2}} &= -\frac{4\pi i}{k}\text{tr}\left(\epsilon^{ABCD}\delta_3\psi_A Y_B^\dagger\psi_C Y_D^\dagger\right), \\ V_{\text{bos2}} &= -\frac{16\pi^2}{3k^2}\text{tr}\left(Y_A^\dagger Y^A Y_B^\dagger Y^B Y_C^\dagger Y^C + Y^A Y_A^\dagger Y^B Y_B^\dagger Y^C Y_C^\dagger \right. \\ &\qquad \left. + Y_A^\dagger Y^B Y_C^\dagger Y^A Y_B^\dagger Y^C - 3Y^A Y_B^\dagger Y^B Y_A^\dagger Y^C Y_C^\dagger\right). \end{aligned} \tag{B.11}$$

From these results we find

$$V_{\rm bos1} + V_{\rm bos2} = V_{\rm bos},$$

where $V_{\rm bos}$ is the bosonic potential of ABJM theory. From (B.1), (B.6), (B.9), and (B.10), we prove that the ABJM action (3.1) is invariant under the supersymmetry transformation rule (3.7) (without $\check{T}$), which can be obtained from (B.2) and (B.7), and their complex conjugates.

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